

JEE MAIN (Session 2) 2023 Paper Analysis

MATHS | 13th April 2023 _ Shift-2



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JEE (Advanced)
9395

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Most Promising **RANKS**
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Percentage (%) Ratio

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AIR-1 to 10
25 Times

AIR-11 to 50
83 Times

AIR-51 to 100
81 Times

JEE MAIN+ADVANCED

AIR-1 to 10
8 Times

AIR-11 to 50
32 Times

AIR-51 to 100
36 Times

Student Qualified
in NEET

(2022)

$4837/5356 = 90.31\%$

(2021)

$3276/3411 = 93.12\%$

Student Qualified
in JEE ADVANCED

(2022)

$1756/4818 = 36.45\%$

(2021)

$1256/2994 = 41.95\%$

Student Qualified
in JEE MAIN

(2022)

$4818/6653 = 72.41\%$

(2021)

$2994/4087 = 73.25\%$



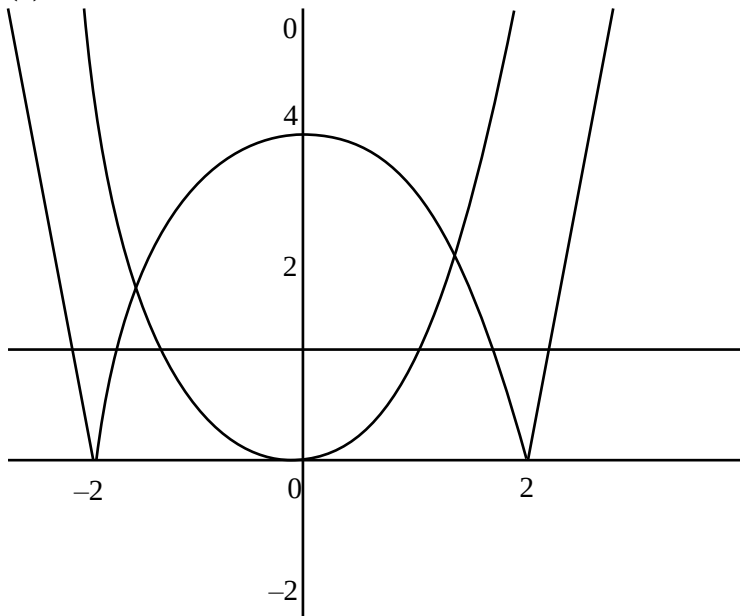
NITIN VIJAY (NV Sir)
Founder & CEO

SECTION - A

1. The area of the region $\{(x, y) : x^2 \leq y \leq |x^2 - 4|, y \geq 1\}$ is:

- (1) $\frac{3}{4}(4\sqrt{2} + 1)$ (2) $\frac{4}{3}(4\sqrt{2} - 1)$ (3) $\frac{3}{4}(4\sqrt{2} - 1)$ (4) $\frac{4}{3}(4\sqrt{2} + 1)$

Sol. (2)



$$\text{Required area} = 2 \left[\int_1^2 \sqrt{y} dy + \int_2^4 \sqrt{4-y} dy \right] = \frac{4}{3} [4\sqrt{2} - 1]$$

2. If $\lim_{x \rightarrow 0} \frac{e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2}}{1 - \cos(2x)} = 17$, then $5a^2 + b^2$ is equal to

- (1) 76 (2) 72 (3) 64 (4) 68

Sol. (4)

$$\lim_{x \rightarrow 0} \frac{e^{ax} - \cos bx - \frac{cxe^{-cx}}{2}}{1 - \cos 2x} = 17$$

On expansion

$$\lim_{x \rightarrow 0} \frac{\left(1 + ax + \frac{(ax)^2}{2!} + \dots\right) - \left(1 - \frac{(bx)^2}{2!} + \dots\right) - \frac{cx}{2} \left(1 - cx + \frac{(cx)^2}{2!}\right)}{\left(\frac{1 - \cos 2x}{(2x)^2}\right) \times (2x)^2} = 17$$

$$\lim_{x \rightarrow 0} \frac{x \left(a - \frac{c}{2}\right) + x^2 \left(\frac{a^2}{2} + \frac{b^2}{2} + \frac{c^2}{2}\right)}{\frac{1}{2}(4x^2)} = 17$$

For limit to be exist

$$a - \frac{c}{2} = 0 \Rightarrow c = 2a$$

$$\Rightarrow \frac{\frac{a^2}{2} + \frac{b^2}{2} + \frac{c^2}{2}}{2} = 17$$

$$\Rightarrow \frac{a^2}{2} + \frac{b^2}{2} + \frac{4a^2}{2} = 34$$

$$\Rightarrow 5a^2 + b^2 = 68$$

3. The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$, is

$$(1) \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$$

$$(2) \frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$$

$$(3) \frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$$

$$(4) \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$$

Sol. (1)

Condition of co-planarity

$$\begin{vmatrix} x_2 - x_1 & a_1 & a_2 \\ y_2 - y_1 & b_1 & b_2 \\ z_2 - z_1 & c_1 & c_2 \end{vmatrix} = 0$$

Where a_1, b_1, c_1 are direction cosine of 1st line and a_2, b_2, c_2 are direction cosine of 2nd line.

Now. Solving options

Point $(-3, 1, 5)$ & point $(-1, 2, 5)$

$$(1) \begin{vmatrix} -3 & 1 & 5 \\ 1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$$

$$= -3(5) - (10) + 5(-1 + 4)$$

$$= -15 - 10 + 15 = -10$$

(2) point $(-1, 2, 5)$

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$$

$$= 3(5) - (10) + 5(1 + 4)$$

$$-25 + 5 = 0$$

(3) point $(-1, 2, 5)$

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 4 \\ -2 & -1 & 0 \end{vmatrix}$$

$$-3(4) - (8) + 5(1 + 4)$$

$$-12 - 8 + 25 = 5$$

(4) point $(-1, 2, 5)$

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ 4 & 1 & 0 \end{vmatrix}$$

$$-3(-5) - (-20) + 5(-1 - 8)$$

$$15 + 20 - 45 = -10$$

4. The plane, passing through the points (0, -1, 2) and (-1, 2, 1) and parallel to the line passing through (5, 1, -7) and (1, -1, -1), also passes through the point

- (1) (0, 5, -2) (2) (-2, 5, 0)
(3) (2, 0, 1) (4) (1, -2, 1)

Sol. (2)

Plane passing through (0, -1, 0) and (-1, 2, 1)

Then vector in plane $\langle -1, 3, -1 \rangle$ vector parallel to plane is $\langle 4, 2, -6 \rangle$

$$\text{Normal vector to plane } (\vec{n}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 4 & 2 & -6 \end{vmatrix}$$

$$= \hat{i}(16) - \hat{j}(10) + \hat{k}(-14)$$

$$\vec{n} = \langle 8, 5, 7 \rangle$$

Equation of plane

$$8(x - 0) + 5(y + 1) + 7(z - 2) = 0$$

$$\Rightarrow 8x + 5y + 7z = 9$$

From given options point (-2, 5, 0) lies on plane.

5. Let for a triangle ABC,

$$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$$

$$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$$

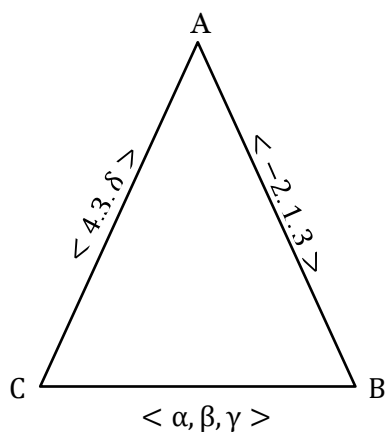
$$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB} \cdot \overrightarrow{CA}$ is equal to

- (1) 108 (2) 60 (3) 54 (4) 120

Sol. (2)

5.



$$\overrightarrow{CA} + \overrightarrow{AB} = \overrightarrow{CB}$$

$$\langle 4, 3, \delta \rangle + \langle -2, 1, 3 \rangle = \overrightarrow{CB}$$

$$\Rightarrow \overrightarrow{CB} = \langle 2, 4, 3 + \delta \rangle$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 3 \\ -4 & -3 & -\delta \end{vmatrix}$$

$$= \hat{i}(9 - \delta) - \hat{j}(2\delta + 12) + \hat{k}(10)$$

$$|\overrightarrow{AB} \times \overrightarrow{AC}|^2 = (9 - \delta)^2 + (2\delta + 12)^2 + (10)^2$$

$$= 5\delta^2 + 30\delta + 325$$

$$\text{Area of } \Delta ABC = 5\sqrt{6}$$

$$\Rightarrow \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}| = 5\sqrt{6}$$

$$\Rightarrow |\overrightarrow{AB} \times \overrightarrow{AC}|^2 = 600$$

$$\Rightarrow 5\delta^2 + 30\delta - 275 = 0$$

$$\Rightarrow \delta^2 + 6\delta - 55 = 0$$

$$\Rightarrow (\delta + 11)(\delta - 5) = 0$$

$$\delta = 5$$

$$\overrightarrow{CB} = \langle 2, 3, 8 \rangle$$

$$\overrightarrow{CB} \cdot \overrightarrow{CA} = \langle 2, 4, 8 \rangle \cdot \langle 4, 3, 5 \rangle$$

$$= 8 + 12 + 40 = 60$$

6. Let for $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $|A| = 2$. If $|2 \text{ adj } (2 \text{ adj } (2A))| = 32^n$, then $3n + \alpha$ is equal to

(1) 10

(2) 9

(3) 12

(4) 11

Sol. (4)

$$|A| = 2$$

$$\text{adj}(kA) = k^{m-1} \text{adj} A \quad \{m = \text{order of matrix}\}$$

$$\text{adj}(2A) = 2^2 \text{adj} A = 4 \text{adj}(A)$$

$$\text{adj}(2 \text{ adj } (2A)) = \text{adj}(8 \text{ adj } A)$$

$$= 8^2 \text{adj adj } (A)$$

$$|2 \text{ adj } 2 \text{ adj } (2A)| = |2^7 \text{adj adj } (A)|$$

$$= (2^7)^3 |A|^{2^2}$$

$$= 2^{21} |A|^4$$

$$= 2^{21} \cdot 2^4$$

$$\Rightarrow 2^{25} = (32)^n$$

$$\Rightarrow 2^{25} = 2^{5n}$$

$$\Rightarrow n = 5$$

$$|A| = 2$$

$$(6-1) - 2(2\alpha-1) + 3(\alpha-3) = 2$$

$$\Rightarrow 5 - 4\alpha + 2 + 3\alpha - 9 = 2$$

$$\Rightarrow \alpha = -4$$

$$3n + \alpha = 11$$

7. The range of $f(x) = 4 \sin^{-1} \left(\frac{x^2}{x^2 + 1} \right)$ is

(1) $[0, \pi)$

(2) $[0, \pi]$

(3) $[0, 2\pi)$

(4) $[0, 2\pi]$

Sol. (3)

$$f(x) = 4 \sin^{-1} \left(\frac{x^2}{1+x^2} \right)$$

$$0 \leq \frac{x^2}{1+x^2} < 1$$

$$\Rightarrow 0 \leq \sin^{-1} \left(\frac{x^2}{1+x^2} \right) < \frac{\pi}{2}$$

$$\Rightarrow 0 \leq 4 \sin^{-1} \left(\frac{x^2}{1+x^2} \right) < 2\pi$$

$$\text{Range : } [0, 2\pi)$$

8. Let $a_1, a_2, a_3, \dots$ be a G. P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to

(1) 2

(2) 3

(3) $3\sqrt{3}$

(4) $2\sqrt{2}$

Sol. (2)

$$a_3 \cdot a_5 = \frac{1}{9}$$

$$\Rightarrow ar^2 \cdot ar^4 = \frac{1}{9}$$

$$\Rightarrow (ar^3)^2 = \frac{1}{9}$$

$$\Rightarrow ar^3 = \frac{1}{3} \quad \dots(i)$$

$$a_6 + a_8 = 2$$

$$\Rightarrow ar^5 + ar^7 = 2$$

$$\Rightarrow ar^3(r^2 + r^4) = 2$$

$$\Rightarrow \frac{1}{3} r^2(1 + r^2) = 2$$

$$\Rightarrow r^2(1 + r^2) = 2 \times 3$$

$$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$$

$$a = \frac{1}{3} \times \frac{1}{r^3}$$

$$= \frac{1}{3} \times \frac{1}{2\sqrt{2}} = \frac{1}{6\sqrt{2}}$$

$$6(a_2 + a_4)(a_4 + a_6)$$

$$\Rightarrow 6(ar + ar^3)(ar^3 + ar^5)$$

$$\Rightarrow 6 \left(\frac{ar^3}{r^2} + \frac{1}{3} \right) \left(\frac{1}{3} + \frac{1}{3} r^2 \right) = 3$$

9. If the system of equations
 $2x + y - z = 5$
 $2x - 5y + \lambda z = \mu$
 $x + 2y - 5z = 7$
 has infinitely many solutions, then $(\lambda + \mu)^2 + (\lambda - \mu)^2$ is equal to
 (1) 904 (2) 916 (3) 912 (4) 920

Sol.

2

$$\Delta = 0$$

$$\Rightarrow \begin{vmatrix} 2 & 1 & -1 \\ 2 & -5 & \lambda \\ 1 & 2 & -5 \end{vmatrix} = 0$$

$$\Rightarrow 2(25 - 2\lambda) - 1(-10 - \lambda) - 1(4 + 5) = 0$$

$$\Rightarrow 51 - 3\lambda = 0$$

$$\Rightarrow \lambda = 17$$

$$\Delta_x = 0$$

$$\begin{vmatrix} 5 & 1 & -1 \\ \mu & -5 & 17 \\ 7 & 2 & -5 \end{vmatrix} = 0$$

$$\Rightarrow 5(25 - 34) - 1(-5\mu - 119) - 1(2\mu + 35) = 0$$

$$\Rightarrow -45 + 5\mu + 119 - 2\mu - 35 = 0$$

$$\Rightarrow 39 + 3\mu = 0 \Rightarrow \mu = -13$$

$$(\lambda + \mu)^2 + (\lambda - \mu)^2 = 4^2 + (30)^2$$

$$= 916$$

10. The statement $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$ is equivalent to _____.

(1) $(\sim p) \vee (\sim q)$

(2) $(\sim p) \wedge (\sim q)$

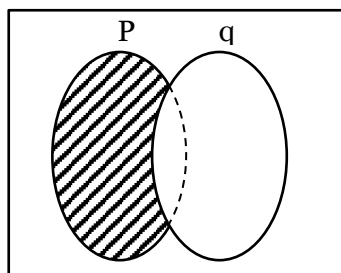
(3) $p \vee (\sim q)$

(4) $p \vee q$

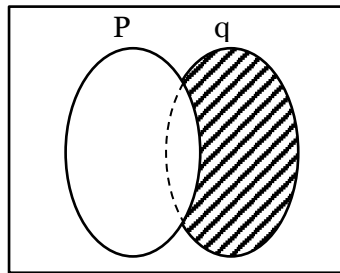
Sol.

(1)

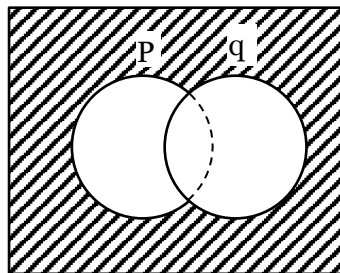
$$(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$$



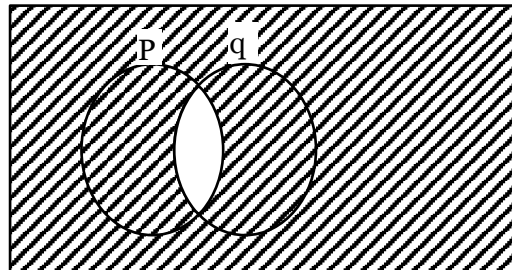
$$p \wedge \sim q \Rightarrow$$



$$\sim p \wedge q =$$



$$\sim p \wedge \sim q =$$



$$(p \wedge \sim q) \vee (\sim p \wedge q) (\sim p \wedge \sim q)$$

$$(\alpha, \beta)$$

$$(\sim p) \vee (\sim q)$$

Plane passing through $(0, -1, 2)$

and $(-1, 2, 1)$

then vector in plane $\langle -1, 3, -1 \rangle$

vector parallel to plane is $\langle 4, 2, -6 \rangle$

normal vector to plane L_2

$$(\vec{n}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 4 & 2 & -6 \end{vmatrix}$$

$$= \hat{i}(-16) - \hat{j}(10) + \hat{k}(-14)$$

$$\vec{n} = \langle 8, 5, 7 \rangle$$

Equation of plane

$$8(x-0) + 5(y+1) + 7(z-2) = 0$$

$$\Rightarrow 8x + 5y + 7z = 9$$

From given options point $(-2, 5, 0)$

lies on plane.

11. Let $S = \{z \in \mathbb{C} : \bar{z} = i(z^2 + \operatorname{Re}(\bar{z}))\}$. Then $\sum_{z \in S} |z|^2$ is equal to

- (1) 4 (2) $\frac{7}{2}$ (3) 3 (4) $\frac{5}{2}$

Sol. (1)

Let $z = x + iy$

$$\bar{z} = i(z^2 + \operatorname{Re}(\bar{z}))$$

$$\Rightarrow x - iy = i(x^2 - y^2 + 2ixy + x)$$

$$\Rightarrow x - iy = -2xy + i(x^2 - y^2 + x)$$

$$x + 2xy = 0 \text{ and } x^2 - y^2 + x + y = 0$$

$$x(1 + 2y) = 0 \text{ and } x^2 - y^2 + x + y = 0$$

If $x = 0$ then $-y^2 + y = 0$

$$\Rightarrow y = 1, 0$$

If $y = \frac{-1}{2}$ then $x^2 - \frac{1}{4} + x - \frac{1}{2} = 0$

$$\Rightarrow x = -\frac{3}{2}, \frac{1}{2}$$

$$= \left\{ 0 + i0, 0 + i, -\frac{3}{2} - \frac{1}{2}i, \frac{1}{2} - \frac{1}{2}i \right\}$$

$$\sum_{z \in S} |z|^2 = 0 + 1 + \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 4$$

12. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$, Then $\alpha^{14} + \beta^{14}$ is equal to

- (1) $-128\sqrt{2}$ (2) $-64\sqrt{2}$ (3) -128 (4) -64

Sol. (3)

$$x^2 - \sqrt{2}x + 2 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$$x = \frac{\sqrt{2} \pm \sqrt{-6}}{2}$$

$$= \sqrt{2} \left(\frac{1 \pm i\sqrt{3}}{2} \right)$$

$$= -\sqrt{2}\omega, -\sqrt{2}\omega^2$$

$$\Rightarrow \alpha = -\sqrt{2}\omega, \beta = -\sqrt{2}\omega^2$$

$$\alpha^{14} + \beta^{14} = 2^7 (\omega^{14} + \omega^{28}) = 2^7 (\omega^2 + \omega) = -128$$

13. Let $|\vec{a}| = 2$, $|\vec{b}| = 3$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$. Then $\left| (\vec{a} + 2\vec{b}) \times (2\vec{a} - 3\vec{b}) \right|^2$ is equal to

- (1) 482 (2) 841 (3) 882 (4) 441

Sol. (3)

$$\cos\left(\frac{\pi}{4}\right) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\vec{a} \cdot \vec{b}}{(2)(3)} \Rightarrow \vec{a} \cdot \vec{b} = 3\sqrt{2}$$

$$\text{Let } \vec{p} = \vec{a} + 2\vec{b}$$

$$\vec{q} = 2\vec{a} - 3\vec{b}$$

$$|\vec{p}|^2 = |\vec{a}|^2 + 4|\vec{b}|^2 + 4(\vec{a} \cdot \vec{b})$$

$$= 4 + 36 + 12\sqrt{2}$$

$$= 40 + 12\sqrt{2}$$

$$|\vec{q}|^2 = 4|\vec{a}|^2 + 9|\vec{b}|^2 - 12(\vec{a} \cdot \vec{b})$$

$$= 16 + 81 - 36\sqrt{2}$$

$$= 97 - 36\sqrt{2}$$

$$\vec{p} \cdot \vec{q} = 2|\vec{a}|^2 - 6|\vec{b}|^2 + \vec{a} \cdot \vec{b}$$

$$= 8 - 54 + 3\sqrt{2}$$

$$= -46 + 3\sqrt{2}$$

$$|\vec{p} \times \vec{q}| = (|\vec{p}| |\vec{q}|)^2 - (\vec{p} \cdot \vec{q})^2$$

$$= (40 + 12\sqrt{2})(97 - 36\sqrt{2}) - (3\sqrt{2} - 46)^2$$

$$= (3016 - 276\sqrt{2}) - (2134 - 276\sqrt{2})$$

$$= 882$$

14. The value of $\frac{e^{-\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} e^{-x} \tan^{50} x dx}{\int_0^{\frac{\pi}{4}} e^{-x} (\tan^{49} x + \tan^{51} x) dx}$ is

(1) 25

(2) 51

(3) 50

(4) 49

Sol. (3)

$$\text{let } I_1 = e^{-\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} e^{-x} \tan^{50} x dx$$

$$I_2 = \int_0^{\frac{\pi}{4}} e^{-x} (\tan^{49} x + \tan^{51} x) dx$$

$$\begin{aligned}
 &= \int_0^{\pi/4} e^{-x} \tan^{49} x (\sec^2 x) dx \\
 &= \left| e^{-x} \frac{\tan^{50} x}{50} \right|_0^{\pi/4} + \frac{1}{50} \int_0^{\pi/4} e^{-x} \tan^{50} x dx \\
 &= \frac{e^{-\pi/4}}{50} + \frac{1}{50} \int_0^{\pi/4} e^{-x} \tan^{50} x dx = \frac{I_1}{50} \\
 &\text{then } \frac{I_1}{I_2} = 50
 \end{aligned}$$

15. The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is

- (1) $\frac{80}{9}$ (2) 8 (3) 9 (4) $\frac{26}{3}$

Sol. (1)

general term for $\left(2x^3 - \frac{1}{3x^2}\right)^5$

$$\begin{aligned}
 T_{r+1} &= {}^5C_r \left(-\frac{1}{3x^2}\right)^r (2x^3)^{5-r} \\
 &= {}^5C_r (-1)^r 3^{-r} 2^{5-r} x^{15-5r}
 \end{aligned}$$

$$15 - 5r = 5 \Rightarrow r = 2$$

$$\text{Coeff. Of } x^5 = {}^5C_2 (-1)^2 3^{-2} 2^3$$

$$\begin{aligned}
 &= 10 \times \frac{1}{9} \times 8 \\
 &= \frac{80}{9}
 \end{aligned}$$

16. The random variable X follows binomial distribution $B(n, p)$, for which the difference of the mean and the variance is 1. If $2P(x=2) = 3P(x=1)$, then $n^2P(X > 1)$ is equal to

- (1) 16 (2) 11 (3) 12 (4) 15

Sol. 2

$$2P(x=2) = 3P(x=1)$$

$$2 \times {}^nC_2 P^2 (1-P)^{n-2} = 3 {}^nC_1 P (1-P)^{n-1}$$

$$\Rightarrow 2 \frac{n(n-1)}{2} \cdot P = 3n(1-P)$$

$$\Rightarrow (n-1)P = 3(1-P) \cdots (i)$$

$$nP - nPq = 1$$

$$\Rightarrow nP - nP(1-p) = 1$$

$$\Rightarrow nP^2 = 1 \Rightarrow n = \frac{1}{p^2}$$

$$\Rightarrow \text{put in equ (i)}$$

$$\left(\frac{1}{p^2} - 1\right)P = 3(1-p)$$

$$\frac{1}{p} - P = 3 - 3P$$

$$\Rightarrow 1 - P^2 = 3P - 3p^2$$

$$\Rightarrow 2P^2 - 3P + 1 = 0$$

$$\Rightarrow 2P^2 - 2P - P + 1 = 0$$

$$\Rightarrow 2P(P-1) - 1(P-1) = 0$$

$$\Rightarrow P = \frac{1}{2}, P = 1 \{\text{Rejected}\}$$

$$n = \frac{1}{(1/2)^2} = 4$$

$$n^2 P(x > 1) = n^2 p(1 - P(x=0) - P(x=1))$$

$$\Rightarrow n^2 P(1 - (1-P)^n - nP(1-P)^{n-1})$$

$$\Rightarrow (4)^2 \left(1 - \left(\frac{1}{2}\right)^4 - 4\left(\frac{2}{2}\right)^4\right)$$

$$\Rightarrow 16 - 1 - 4 = 11$$

17. Let the centre of a circle C be (α, β) and its radius $r < 8$. Let $3x + 4y = 24$ and $3x - 4y = 32$ be two tangents and $4x + 3y = 1$ be a normal to C. Then $(\alpha - \beta + r)$ is equal to

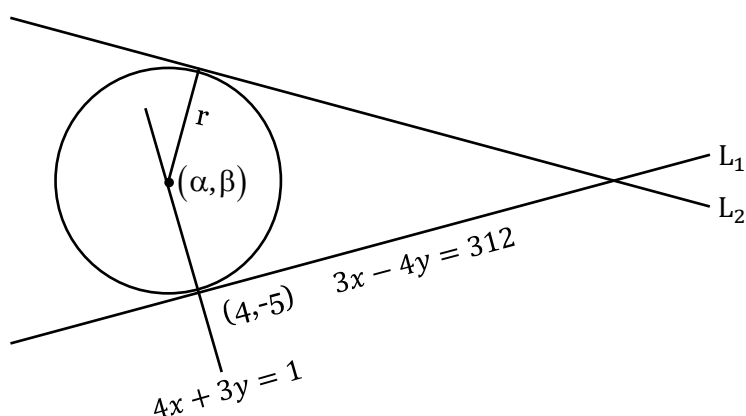
(1) 5

(2) 6

(3) 7

(4) 9

Sol. (3)



Centre lies on normal

$$\Rightarrow 4\alpha + 3\beta = 1$$

Distance of (α, β) from L_1 and L_2 are equal

$$\left| \frac{3\alpha + 4\beta - 24}{5} \right| = \left| \frac{3\alpha - 4\beta - 32}{5} \right|$$

$$3\alpha + 4\beta - 24 = -3\alpha + 4\beta + 32$$

$$\Rightarrow 6\alpha = 56$$

$$\Rightarrow \alpha = \frac{28}{3}, \beta = \frac{-109}{3}$$

$$r = \sqrt{\left(\frac{28}{3} - 4\right)^2 + \left(\frac{-109}{3} + 5\right)^2} > 8$$

(reject)

$$3\alpha + 4\beta - 24 = -3\alpha - 4\beta - 32$$

$$8\beta = -8$$

$$\beta = -1, \alpha = 1$$

$$\gamma = \sqrt{(4-1)^2 + (-5+1)^2} = 5$$

$$\alpha - \beta + \gamma = 7$$

18. Let N be the foot of perpendicular from the point P (1, -2, 3) on the line passing through the points (4, 5, 8) and (1, -7, 5). Then the distance of N from the plane $2x - 2y + z + 5 = 0$ is

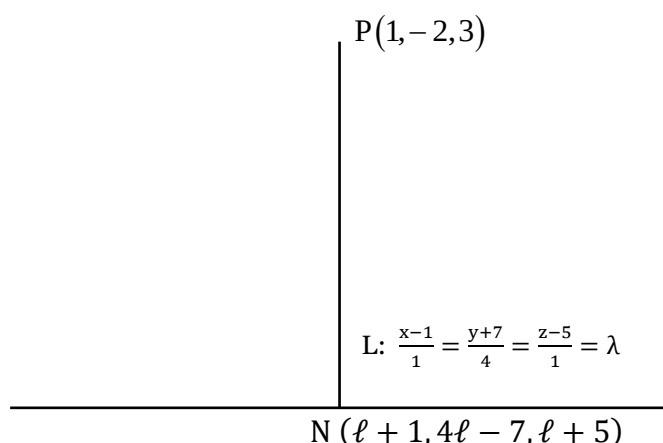
(1) 6

(2) 7

(3) 9

(4) 8

Sol. (2)



$$\overrightarrow{PN} = \langle \lambda, 4\lambda - 5, \lambda + 2 \rangle$$

$$\overrightarrow{PN} \cdot \langle 1, 4, 1 \rangle = 0$$

$$\Rightarrow \lambda + 16\lambda - 20 + \lambda + 2 = 0$$

$$\Rightarrow \lambda = 1$$

$$N(2, -3, 6)$$

Distance of N from $2x - 2y + z + 5 = 0$ is

$$d = \frac{|2(2) - 2(-3) + 6 + 5|}{\sqrt{2^2 + (-2)^2 + 1^2}}$$

$$= \frac{|21|}{3} = 7$$

19. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is

(1) 328

(2) 327

(3) 324

(4) 326

Sol. (2)

$$A \overbrace{\square \square \square \square \square}^{5!} = 120$$

$$\boxed{D} \overbrace{\boxed{} \boxed{} \boxed{} \boxed{} \boxed{}}^{5!} = 120$$

$$\boxed{M} \boxed{A} \overbrace{\boxed{} \boxed{} \boxed{} \boxed{}}^{4!} = 24$$

$$\boxed{M} \boxed{D} \overbrace{\boxed{} \boxed{} \boxed{} \boxed{}}^{4!} = 24$$

$$\boxed{M} \boxed{N} \overbrace{\boxed{} \boxed{} \boxed{} \boxed{}}^{4!} = 24$$

$$\boxed{M} \boxed{O} \boxed{A} \overbrace{\boxed{} \boxed{} \boxed{}}^{3!} = 6$$

$$\boxed{M} \boxed{O} \boxed{D} \overbrace{\boxed{} \boxed{} \boxed{}}^{3!} = 6$$

$$\boxed{M} \boxed{O} \boxed{N} \boxed{A} \overbrace{\boxed{} \boxed{}}^{2!} = 2$$

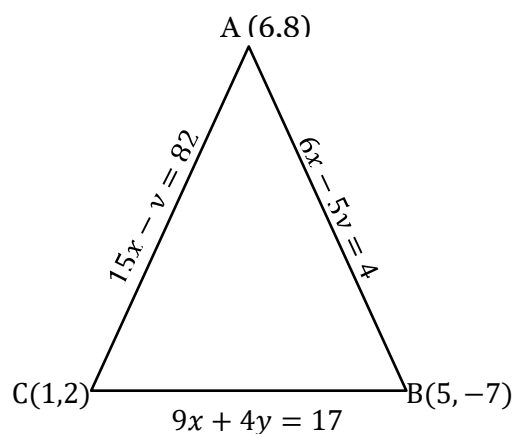
$$\boxed{M} \boxed{O} \boxed{N} \boxed{D} \boxed{A} \boxed{Y} = 1$$

$$\begin{aligned} \text{Rank} &= 120 + 120 + 24 + 24 + 24 + 6 + 6 + 2 + 1 \\ &= 327 \end{aligned}$$

20. Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82$, $6x - 5y = -4$ and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation

(1) $x^2 - 13x + 42 = 0$ (2) $x^2 - 10x + 25 = 0$ (3) $x^2 - 7x + 12 = 0$ (4) $x^2 - 14x + 48 = 0$

Sol. (1)



$$\text{Centroid } (\alpha, \beta) = \left(\frac{6+1+5}{3}, \frac{8-7+2}{3} \right) = (4, 1)$$

$$\alpha + 2\beta = 4 + 2 = 6$$

$$2\alpha - \beta = 8 - 1 = 7$$

Quadratic equation

$$x^2 - (6+7)x + (6 \times 7) = 0$$

$$\Rightarrow x^2 - 13x + 42 = 0$$

SECTION - B

21. Let $A = \{-4, -3, -2, 0, 1, 3, 4\}$ and $R = \{(a, b) \in A \times A : b = |a| \text{ or } b^2 = a + 1\}$ be a relation on A . Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is

Sol. (7)
 $R = \{(-4, 4), (-3, 3), (3, -2), (0, 1), (0, 0), (1, 1), (4, 4), (3, 3)\}$
 For reflexive, add $\Rightarrow (-2, -2), (-4, -4), (-3, -3)$
 For symmetric, add $\Rightarrow (4, -4), (3, -3), (-2, 3), (1, 0)$

22. Let $f_n = \int_0^{\pi/2} \left(\sum_{k=1}^n \sin^{k-1} x \right) \left(\sum_{k=1}^n (2k-1) \sin^{k-1} x \right) \cos x \, dx, n \in \mathbb{N}$. Then $f_{21} - f_{20}$ is equal to _____

Sol. (41)

$$f_n = \int_0^{\pi/2} \left(\sum_{k=1}^n \sin^{k-1} x \right) \left(\sum_{k=1}^n (2k-1) \sin^{k-1} x \right) \cos x \, dx$$

$$\sin x = t$$

$$\cos x \, dx = dt$$

$$f_n = \int_0^1 \left(\sum_{k=1}^n t^{k-1} \right) \left(\sum_{k=1}^n (2k-1) t^{k-1} \right) dt$$

$$= \int_0^1 (1 + t + t^2 + \dots + t^{n-1}) (1 + 3t + 5t^2 + \dots + (2n-1)t^{n-1}) dt$$

$$f_{n+1} = \int_0^1 \left(\sum_{k=1}^{n+1} t^{k-1} \right) \left(\sum_{k=1}^{n+1} (2k-1) t^{k-1} \right) dt$$

$$= \int_0^1 (1 + t + t^2 + \dots + t^n) (1 + 3t + 5t^2 + \dots + (2n+1)t^n) dt$$

$$= \int_0^1 (1 + t + t^2 + \dots + t^{n-1}) (1 + 3t + 5t^2 + \dots + (2n-1)t^{n-1}) dt$$

$$+ \int_0^1 (1 + 3t + 5t^2 + \dots + (2n+1)t^n) t^n dt$$

$$+ \int_0^1 (1 + t + t^2 + \dots + t^{n-1}) (2n+1)t^n dt$$

$$f_{n+1} - f_n = \int_0^1 (1 + 3t + 5t^2 + \dots + (2n+1)t^n) t^n dt$$

$$+ \int_0^1 (1 + t + t^2 + \dots + t^{n-1}) ((2n+1)t^n) dt$$

$$\text{put } n=20$$

$$f_{21} - f_{20} = \int_0^1 (1 + 3t + 5t^2 + \dots + 41t^{20}) t^{20} dt + \int_0^1 (1 + t + t^2 + \dots + t^{19}) (41t^{20}) dt$$

$$= \left(\frac{1}{21} + \frac{3}{22} + \frac{5}{23} + \dots + \frac{39}{40} + \frac{41}{41} \right) + \left(\frac{41}{21} + \frac{41}{22} + \frac{41}{40} \right)$$

$$= \frac{1+41}{21} + \frac{3+41}{22} + \dots + \frac{39+41}{40} + 1 = 40 + 1 = 41$$

23. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{4x}{(x^2-1)}y = \frac{x+2}{(x^2-1)^{5/2}}, x > 1$ such that

$$y(2) = \frac{2}{9} \log_e(2 + \sqrt{3}) \text{ and } y(\sqrt{2}) = \alpha \log_e(\sqrt{\alpha} + \beta) + \beta - \sqrt{\gamma}, \alpha, \beta, \gamma \in \mathbb{N}, \text{ then } \alpha\beta\gamma \text{ is equal to } \underline{\hspace{2cm}}.$$

Sol. (6)

given differential equation $\frac{dy}{dx} + \frac{4x}{(x^2-1)}y = \frac{x+2}{(x^2-1)^{5/2}}$ is linear D.E.

$$\text{I.F.} = \int \frac{4x}{x^2-1} dx = {}_e 2 \ln(x^2-1) = {}_e \ln(x^2-1)^2 = (x^2-1)^2$$

$$y(x^2-1)^2 = \int \frac{x+2}{(x^2-1)^{5/2}} (x^2-1)^2 dx$$

$$= \int \frac{x}{\sqrt{x^2-1}} dx + \int \frac{2dx}{\sqrt{x^2-1}}$$

$$= \sqrt{x^2-1} + 2 \ln[x + \sqrt{x^2-1}] + C$$

$$\text{put } y(2) = \frac{2}{9} \ln(2 + \sqrt{3})$$

$$\frac{2}{9} \ln(2 + \sqrt{3})(9) = \sqrt{3} + 2 \ln[2 + \sqrt{3}] + C$$

$$= C = -\sqrt{3}$$

$$\text{put } x = \sqrt{2}$$

$$y = 1 + 2 \ln[\sqrt{2} + 1] - \sqrt{3}$$

$$\alpha = 2, \beta = 1 = \gamma = 3$$

$$\alpha\beta\gamma = 2(1)(3) = 6$$

24. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is _____.

Sol. 16

a	b	2
---	---	---

$$(a,b) = (1,3), (3,1), (2,2), (2,5), (5,2), (3,4), (4,3), (5,5)$$

= 8 numbers

a	b	4
---	---	---

$$(a,b) = (1,1), (1,4), (4,1), (2,3), (3,2)$$

$$(4,4), (3,5), (5,3) = 8 \text{ numbers}$$

$$\text{total } 8 + 8 = 16$$

25. The remainder, when 7^{103} is divided by 17, is _____.

Sol. 12

$$7^{103} = 7 \cdot 7^{102}$$

$$= 7(7^2)^{51}$$

$$= 7(51 - 2)^{51} \rightarrow \text{remainder} = 7(-2)^{51}$$

$$= 7(2^3)(16)^{12} = -56(17 - 1)^{12} \rightarrow \text{remainder} = -56(-1)^{12}$$

$$\text{Remainder} = -56 + 17k$$

$$= -56 + 68$$

$$= 12$$

26. Let $f(x) = \sum_{k=1}^{10} kx^k$, $x \in \mathbb{R}$ If $2f(2) - f'(2) = 119(2)^n + 1$ then n is equal to _____

Sol. 10

$$f(x) = \sum_{k=1}^{10} kx^k$$

$$\Rightarrow f(x) = x + 2x^2 + 3x^3 + \dots + 9x^9 + 10x^{10} \text{---(i)}$$

$$xf(x) = x^2 + 2x^3 + \dots + 9x^{10} + 10x^{11} \text{---(ii)}$$

$$\text{"(i) - (ii)"}$$

$$f(x)(1-x) = x + x^2 + x^3 + \dots + x^{10} - 10x^{11}$$

$$f(x)(1-x) = \frac{x(1-x^{10})}{1-x} - 10x^{11}$$

$$f(x) = \frac{x(1-x^{10})}{(1-x)^2} - \frac{10x^{11}}{(1-x)}$$

$$f(2) = 2 + g(2)^{11}$$

$$(1-x)^2 f(x) = x(1-x^{10}) - 10x^{11}(1-x)$$

diff. w.r.t. x

$$(1-x)^2 f'(x) + f(x)2(1-x)(-1)$$

$$= x(-10x^9) + (1-x^{10}) - 10x^{11}(-1) - (1-x)(110)x^{10}$$

put $x = 2$

$$f'(2) + f(2)(2) = -10(2)^{10} + 1 - 2^{10} + 10(2)^{11} - 110(2)^{10} + 110(2)^{11}$$

$$= (-121)2^{10} + (120)2^{11} + 1$$

$$= 2^{10}(240 - 121) + 1$$

$$= 119(2)^{10} + 1$$

$$n = 10$$

27. For $x \in (-1, 1]$, the number of solutions of the equation $\sin^{-1} x = 2 \tan^{-1} x$ is equal to ____.

Sol. 2

$$\sin^{-1} x = 2 \tan^{-1} x$$

$$\sin^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$\Rightarrow x = \frac{2x}{1+x^2}$$

$$\Rightarrow x \left(\frac{2}{1+x^2} - 1 \right) = 0$$

$$\Rightarrow x = 0, 1, -1 \text{ but } -1 \text{ is not included.}$$

Answer 2 solutions

28. The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively, Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is ____.

Sol. 269

$$\text{Mean} = \frac{\sum x_i}{10}$$

$$\Rightarrow 50 = \frac{\sum x_i}{10}$$

$$\Rightarrow \sum x_i = 500$$

$$\text{correct } \sum x_i = 500 - 45 - 50 + 20 + 25 = 450$$

$$\sigma^2 = \frac{\sum x_i^2}{10} - \left(\bar{x} \right)^2$$

$$\Rightarrow 144 = \frac{\sum x_i^2}{10} - 2500$$

$$\Rightarrow \sum x_i^2 = 26440$$

$$\text{correct } \sum x_i^2 = 26440 - (45)^2 - (50)^2 + (20)^2 + (25)^2$$

$$= 26440 - 2025 - 2500 + 400 + 625$$

$$= 22940$$

$$\sigma^2 = \frac{\text{correct } \sum x_i^2}{10} - \left(\frac{\text{correct } \sum x_i}{10} \right)^2$$

$$= \frac{22940}{10} - \left(\frac{450}{10} \right)^2 = 2294 - 2025$$

$$= 269$$

29. The foci of a hyperbola are $(\pm 2, 0)$ and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line $2x + 3y = 6$, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x and y - axes are a and b respectively, then $|6a| + |5b|$ is equal to ____.

Sol. 12

$$2ae = 4$$

$$2a \left(\frac{3}{2} \right) = 4$$

$$\Rightarrow a = \frac{4}{3}$$

$$e^2 = 1 + \frac{b^2}{a^2}$$

$$\Rightarrow \frac{9}{4} = 1 + b^2 \left(\frac{9}{16} \right)$$

$$\Rightarrow b^2 = \left(\frac{5}{4} \right) \left(\frac{16}{9} \right) = \frac{20}{9}$$

$$\text{slope of tangent } m = \frac{3}{2}$$

equation of tangent is

$$y = mx \pm \sqrt{a^2 m^2 - b^2}$$

$$y = \frac{3}{2}x \pm \sqrt{\frac{16}{9} \left(\frac{9}{4} \right) - \frac{20}{9}}$$

$$\Rightarrow y = \frac{3x}{2} \pm \frac{4}{3}$$

$$y = 0 \Rightarrow a = \pm \frac{8}{9}$$

$$x = 0 \Rightarrow b = \pm \frac{4}{3}$$

$$|6a| + |5b| = \frac{16}{3} + \frac{20}{3} = 12$$

30. Let $[\alpha]$ denote the greatest integer $\leq \alpha$. Then $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \dots + [\sqrt{120}]$ is equal to ____.

Sol. **825**

$$S = [\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \dots + [\sqrt{120}]$$

$$[\sqrt{1}] \rightarrow [\sqrt{3}] = 1 \times 3$$

$$[\sqrt{4}] \rightarrow [\sqrt{8}] = 2 \times 5$$

$$[\sqrt{9}] \rightarrow [\sqrt{15}] = 3 \times 7$$

$\vdots$

$$[\sqrt{100}] \rightarrow [\sqrt{120}] = 10 \times 21$$

$$S = 1 \times 3 + 2 \times 5 + 3 \times 7 + \dots + 10 \times 21$$

$$= \sum_{r=1}^{10} r(2r+1)$$

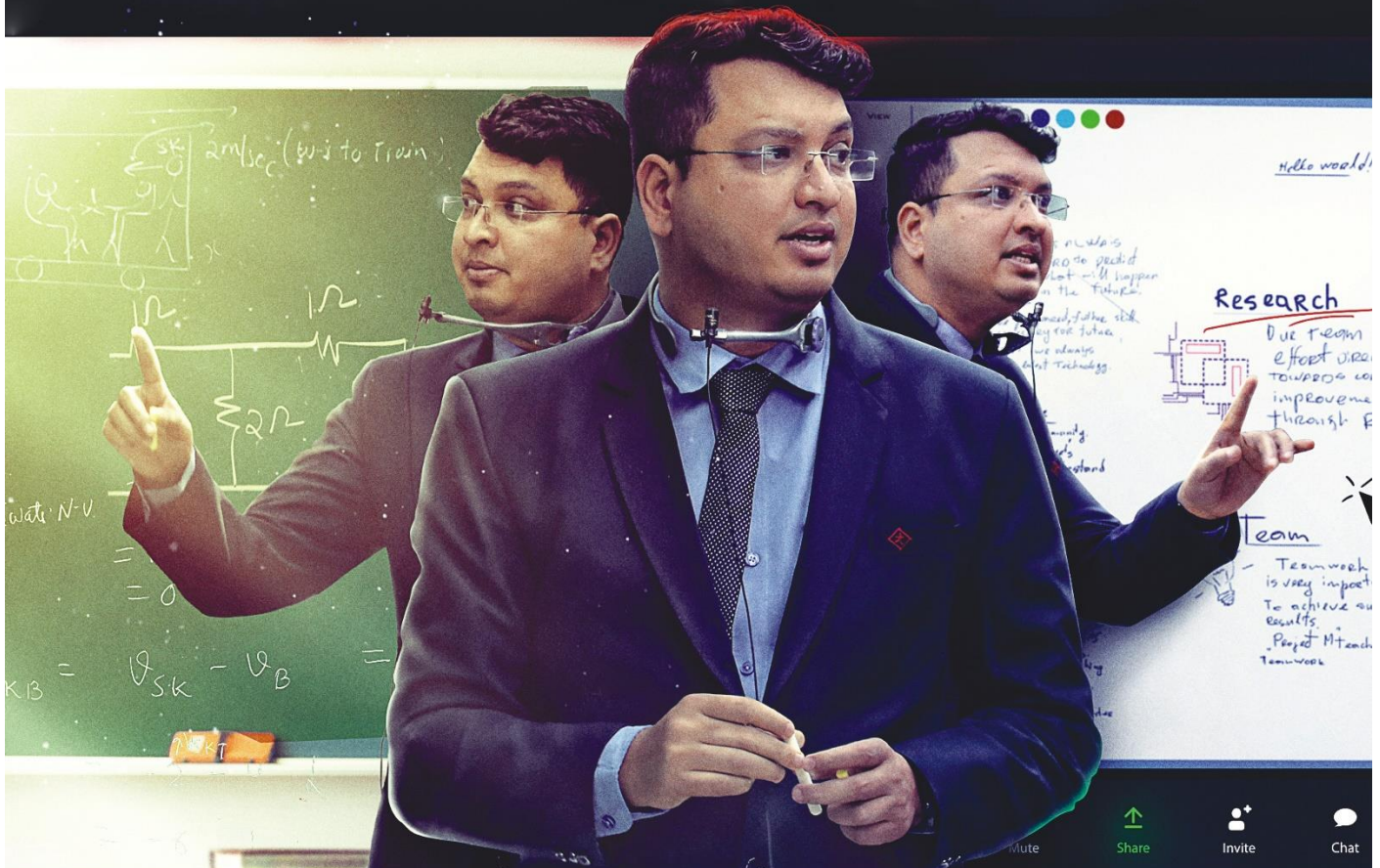
$$= 2 \sum_{r=1}^{10} r^2 + \sum_{r=1}^{10} r$$

$$= \frac{2 \times 10 \times 11 \times 21}{6} + \frac{10 \times 11}{2}$$

$$= 770 + 55$$

$$= 825$$

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